Mathematical Modeling of the Opioid Epidemic
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Abstract
We developed a mathematical model describing opioid addiction and recovery in Allegheny County. Our model tracks the number of susceptible individuals, prescribed opioid users, addicted users, and individuals receiving addiction treatment over time. Individuals who enter addiction treatment are placed in one of four recovery classes. Rates of movement into and out of each of the four recovery classes are estimated from data provided by the Allegheny County Department of Human Services. Model simulations demonstrate the impact of diverse treatment options and the opioid prescription rate on overdose fatalities.

Background
Quick Facts: The Deadly Spread of the Opioid Epidemic
4x the number of overdose deaths in 2018 compared to end of the 20th century
70% of overdose deaths in 2018 involved an opioid
128 people die everyday from an overdose

Prescription Opioids


Model

Variables and Parameters
The following variables track the proportion of the population in each of the seven classes over time.

\[ S(t) \]  (Susceptible): Individuals who are not using opioids or in treatment for addiction

\[ P(t) \]  (Prescribed): Individuals who have been prescribed opioids

\[ A(t) \]  (Addicted): Individuals who are addicted to opioids

\[ R(t) \]  (Recovery): Individuals who are in one of the four sub classes below receiving some form of addiction treatment

- \[ R_1(t) \]  – Assisted with Pharmacuetics
- \[ R_2(t) \]  – Inpatient
- \[ R_3(t) \]  – Outpatient
- \[ R_4(t) \]  – Residential Housing

Table 1. Parameter values for treatment classes.

<table>
<thead>
<tr>
<th>Description</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( R_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment entry rate, ( \zeta )</td>
<td>0.034685</td>
<td>0.200436</td>
<td>1.302917</td>
<td>0.057650</td>
</tr>
<tr>
<td>Successful treatment rate, ( \theta )</td>
<td>0.816445</td>
<td>0.188742</td>
<td>0.562118</td>
<td>0.713349</td>
</tr>
<tr>
<td>Relapse rate, ( \sigma )</td>
<td>0.583296</td>
<td>1.762260</td>
<td>0.843970</td>
<td>0.673344</td>
</tr>
<tr>
<td>Death rate, ( \mu )</td>
<td>0.003759</td>
<td>0.008344</td>
<td>0.004232</td>
<td>0.006610</td>
</tr>
</tbody>
</table>

Results

Statistical software R was used to solve equations (1) – (7) with the following initial conditions:

\[ A(0) = R_2(0) = R_3(0) = R_4(0) = 0 \]

Figure 2. Model solutions show the values of \( S, P, A, R_1, R_2, R_3 \) and \( R_4 \) over time.

Long-Term Impact of Decreasing Opioid Prescriptions
Model simulations were performed to determine the cumulative number of opioid overdose deaths for different prescription rates. Results show that even a large reduction in prescription rates still yields an increasing number of overdose deaths.

Discussion

The results from our model can be implemented to devise an optimal treatment plan in Allegheny County:

- Reductions in prescription rate correspond to reductions in overdose deaths. However, the epidemic still exists.
- Outpatient treatment services are an essential component in any successful plan to minimize the impact of the opioid epidemic.
- These results could be attributed to the willingness of addicts to enroll in outpatient programs as opposed to residential or in-patient programs.

Future Work

Future studies may include model parameters that vary by location within Allegheny County.

Figure 3. Impact of decreasing the opioid prescription rate on overdose deaths over a 15-year period.

Impact of Different Treatment Options on Deaths

Model simulations were performed for different combinations of treatment options. The graph below indicates that the \( R_1 \) treatment class (Outpatient) is critical in reducing overdose deaths.

Figure 4. Bar plot shows the average annual overdose death rate in Allegheny county predicted by the model for different combinations of treatment options.

References


Acknowledgements

Special thank you to Dr. Benedict Kolber of the Department of Biological Sciences and Peter Jern from the Allegheny County DHS for sharing their expertise on the opioid epidemic and assisting with our research.

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